A Primer on Structural Vibrations

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Topics

- Fundamental Concepts
- Common Sources of Dynamic Excitation
- Human Perception of Vibrations
- Measurement
- Vibration Case Studies
What is vibration?

Repetitive motion of a body about an equilibrium position.

Animations courtesy O. Alexandrov
How can vibration be a problem?

• Human Perception
• Serviceability / Operational Issues
• Damage Accumulation / Fatigue
• Overload / Inelastic Behavior
Fundamentals
Analytical Approach

Idealized Structure

Equivalent Spring-Mass-Damper
Single Degree of Freedom (SDOF) Model

\[ k \cdot x(t) + c \cdot \dot{x}(t) + m \cdot \ddot{x}(t) = f(t) \]

**Structural Properties**
- \( m = \text{Mass} \)
- \( c = \text{Damping} \)
- \( k = \text{Stiffness} \)

- \( x(t) = \text{Displacement} \)
- \( \dot{x}(t) = \text{Velocity} \)
- \( \ddot{x}(t) = \text{Acceleration} \)

\[ \sum F = ma \quad \Rightarrow \quad f(t) - cx(t) - kx(t) = m\ddot{x}(t) \]
Rearranging terms gives SDOF equation of motion (EOM):

\[ m\ddot{x}(t) + c\dot{x}(t) + kx(t) = f(t) \]

Related to Structural Properties

Dependent on Excitation Source

Free Vibration \((f(t) = 0)\)

*Natural response to an impact or initial displacement*

Forced Vibration \((f(t) \neq 0)\)

*Response to external time varying force*
Manipulating the EOM introduces important properties:

\[ m[\ddot{x}(t) + 2\zeta\omega_n \dot{x}(t) + \omega_n^2 x(t)] = f(t) \]

**Natural Frequency:**
\[ \omega_n = \sqrt{\frac{k}{m}} \quad \rightarrow \quad f_n = \frac{\omega_n}{2\pi} \]

<table>
<thead>
<tr>
<th>typ. units: ( \text{radians per second} )</th>
<th>typ. units: ( \text{Hertz (Hz)} )</th>
<th>typ. units: ( \text{seconds} )</th>
</tr>
</thead>
</table>

**Period:**
\[ T_n = \frac{2\pi}{\omega_n} \]
Manipulating the EOM introduces important properties:

\[ m[\ddot{x}(t) + 2\zeta\omega_n \dot{x}(t) + \omega_n^2 x(t)] = f(t) \]

**Damping Ratio:**

\[ \zeta = \frac{c}{2\sqrt{km}} \]

Also called percent of *critical damping*. 
SDOF Free Vibration

Natural response to an initial velocity and/or displacement (e.g. “plucking” a guitar string)

\[ x(0) = x_0 \quad \dot{x}(0) = \dot{x}_0 \quad \text{Initial Conditions Only} \]

\[ f(t) = 0 \quad \text{No Time Varying Excitation} \]

EOM becomes:

\[ \ddot{x}(t) + 2\zeta\omega_n \dot{x}(t) + \omega_n^2 x(t) = 0 \]
Undamped Free Vibration

Special (not physically realistic) case where $\zeta = 0$.

$$\ddot{x}(t) + \omega_n^2 x(t) = 0$$

Solving EOM:

$$x(t) = A \cos(\omega_n t - \phi)$$

Amplitude:

$$A = \sqrt{x_0^2 + \left(\frac{\dot{x}_0}{\omega_n}\right)^2}$$

Phase Angle:

$$\phi = \tan^{-1}\left(\frac{\dot{x}_0}{x_0 \omega_n}\right)$$
Undamped Free Vibration

Example: Response with varying stiffness

<table>
<thead>
<tr>
<th>Mass</th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m$</td>
<td>$m$</td>
<td>$m$</td>
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<table>
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<tr>
<th>Stiffness</th>
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<tr>
<td>$k$</td>
<td>$4k$</td>
<td>$9k$</td>
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<table>
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<th>Natural Frequency</th>
<th>A</th>
<th>B</th>
<th>C</th>
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<tr>
<td>$\omega_n$</td>
<td>$2\omega_n$</td>
<td>$3\omega_n$</td>
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</table>

<table>
<thead>
<tr>
<th>Natural Period</th>
<th>A</th>
<th>B</th>
<th>C</th>
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<tbody>
<tr>
<td>$T_n$</td>
<td>$T_n/2$</td>
<td>$T_n/3$</td>
<td></td>
</tr>
</tbody>
</table>

Animation courtesy of Dr. Dan Russell, Grad. Prog. Acoustics, Penn State
Damped Free Vibration

\[ \ddot{x}(t) + 2\zeta \omega_n \dot{x}(t) + \omega_n^2 x(t) = 0 \]

**Super-Critical Damping:** \( \zeta > 1 \)
Mass will not oscillate – exponentially returns to equilibrium

**Critical Damping:** \( \zeta = 1 \)
System will return to equilibrium in shortest possible time
Damped Free Vibration

\[ \ddot{x}(t) + 2\zeta\omega_n \dot{x}(t) + \omega_n^2 x(t) = 0 \]

**Sub-Critical Damping:** \( \zeta < 1 \)

System will oscillate with exponential decay

Solving EOM:

\[ x(t) = Ae^{-\zeta\omega_n t} \cos(\omega_d t - \phi) \]

Damped Natural Frequency:

\[ \omega_d = \omega_n \sqrt{1 - \zeta^2} \]
Undamped vs. Damped Free Vibration
Undamped vs. Damped Free Vibration

Animation courtesy of Dr. Dan Russell, Grad. Prog. Acoustics, Penn State
SDOF Forced Vibration

Periodic vs. Non-Periodic Excitation

Deterministic vs. Random Excitation
SDOF Harmonic Excitation

\[ m[\ddot{x}(t) + 2\zeta\omega_n\dot{x}(t) + \omega_n^2x(t)] = f_o \sin(\omega t) \]

Solving EOM:

\[ x(t) = \frac{(f_o/k)}{\sqrt{(1 - \beta^2)^2 + (2\zeta\beta)^2}} \sin(\omega t - \phi) \]

\[ \beta = \frac{\omega}{\omega_n} \quad \phi = \tan^{-1}\left(\frac{2\zeta\beta}{1 - \beta^2}\right) \]

Harmonic excitation results in harmonic response at excitation frequency, \( \omega \)
Dynamic Amplification Factor (DAF): 
Ratio of amplitude of dynamic response to amplitude of static response

\[ x_{\text{static}} = \frac{f_0}{k} \]

\[ DAF = \frac{|x(t)|}{x_{\text{static}}} = \frac{|x(t)|}{\frac{f_0}{k}} \]

\[ DAF = \frac{1}{\sqrt{(1 - \beta^2)^2 + (2\zeta\beta)^2}} \]
SDOF Harmonic Excitation

Resonance occurs when\n\[ \omega = \omega_n \text{ or } \beta = 1 \]
\( (DAF = 1/2\zeta) \)

\[ DAF \rightarrow \infty \text{ for } \zeta = 0 \text{ and } \beta = 1 \]
SDOF Resonance Demonstration
Multiple Degree of Freedom Systems
Sources of Vibration
Sources of Vibration: Flow

Examples:
- Wind
- Waves
- Hydraulics
- Process piping

Maximum response
\[ x_{max} = \mu_x + 3\sigma_x \]

\[ x(t) \]
\[ t \]
\[ S_x(\omega) \]
\[ \omega_0 \]
\[ \omega \]

\[ \sigma_x^2 \]

\[ \text{Wind} \]
- Random
- Mean + Fluctuating components
Sources of Vibration: Ground Motion

Examples:
- Earthquake
- Construction
- Mining
- Traffic

**EARTHQUAKE**
- Random, non-periodic

- Inertial force (ground acceleration)

\[
m(x(t) + \ddot{z}(t)) + c\dot{x} + kx = 0
\]

\[
m\ddot{x} + c\dot{x} + kx = -m\ddot{z}
\]

\[
\dot{x} + 2\zeta\omega_n\dot{x} + \omega_n^2x = -\ddot{z}
\]
Sources of Vibration: Ground Motion

Inertial Force Analogy

Acceleration

Deceleration (Braking)
CONSTRUCTION

- Empirical standards
  - Based on measured ground motion, not measured structural response
- Peak Particle Velocity (PPV) is key parameter
  - Velocity correlates best with vibration induced strain/damage

Sources of Vibration: Ground Motion

Frequency dependent criteria based on a 5% probability of inducing threshold damage
### Sources of Vibration: Ground Motion

<table>
<thead>
<tr>
<th>Vibration or response source</th>
<th>Magnitude</th>
<th>Equivalent ground vibration velocity, in/s</th>
<th>References</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temperature, outside</td>
<td>$\Delta 10^\circ F$</td>
<td>1.0 - 3.2</td>
<td>Stagg (1984)</td>
</tr>
<tr>
<td></td>
<td>$\Delta 10^\circ F$</td>
<td>0.5 - 1.7</td>
<td>Siskind (1996)</td>
</tr>
<tr>
<td></td>
<td>$\Delta 18^\circ F$</td>
<td>&gt;0.34</td>
<td>White (1993)</td>
</tr>
<tr>
<td>Temperature and humidity cycles</td>
<td>not specified</td>
<td>1.75 - 5.0</td>
<td>Fang (1976)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.75 - 2.6</td>
<td>Dowding (1996)</td>
</tr>
<tr>
<td>Humidity</td>
<td>10 pct</td>
<td>1.0 - 2.4</td>
<td>Stagg (1984)</td>
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<tr>
<td>Wind</td>
<td>20 mph</td>
<td>0.6 - 2.6</td>
<td>Stagg (1984)</td>
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<tr>
<td></td>
<td>50 mph</td>
<td>1.1 - 6.7</td>
<td>Sutherland (1968)</td>
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<td>Traffic</td>
<td>4-t truck driving over a 1-in plank at 63 ft</td>
<td>$=0.24$</td>
<td>Thoenen (1942)</td>
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<tr>
<td></td>
<td>not specified</td>
<td>0.04 - 0.20</td>
<td>Fang (1976)</td>
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<tr>
<td>Human activity:</td>
<td>slamming front door</td>
<td>0.15 - 1.9</td>
<td>Stagg (1984)</td>
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<tr>
<td></td>
<td>closing door</td>
<td>0.35 - 0.50</td>
<td>Aimone (1987)</td>
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<tr>
<td></td>
<td>walking</td>
<td>0.1</td>
<td>Aimone (1987)</td>
</tr>
<tr>
<td></td>
<td>pushing on wall</td>
<td>0.025 - 0.36</td>
<td>Fang (1976)</td>
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<tr>
<td></td>
<td>pushing on wall</td>
<td>0.6 - 2.4</td>
<td>White (1993)</td>
</tr>
<tr>
<td></td>
<td>jumping &amp; walking</td>
<td>0.10 - 0.50</td>
<td>Stagg (1984)</td>
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<tr>
<td></td>
<td>jumping</td>
<td>0.15 - 0.9</td>
<td>White (1993)</td>
</tr>
<tr>
<td></td>
<td>walking (long span floor response)</td>
<td>0.16 - 0.74</td>
<td>Dowding (1996)</td>
</tr>
</tbody>
</table>
Examples:
- Combustible dust
- Vapor cloud
- Boiling Liquid
- Expanding Vapor
- High explosives

Sources of Vibration: Explosions

• Transient

\[
\int_{t_0}^{t} p(t) dt = -p_{so} t_d
\]

Idealized Shock Wave

\[ t' = t - t_d \]
Sources of Vibration: Equipment & People

Examples:

*Equipment*
- Rotating process machinery
- HVAC systems
- Washing machines

*People*
- Walking / foot-fall
- Dance / exercise

- Periodic, narrowband
Other quick tidbits...
Expectation & perception depend on situation
  – e.g. attending a football stadium vs. sitting in a quiet home
Perception depend on both frequency and amplitude of motion
  – Humans are more sensitive in 4-7 Hz range
Greater sensitivity to intermittent vibrations than continuous
  – e.g. quarry blasting vs. railway vibrations
Health effects depend on axis of vibration and exposure (dose)

- ISO 2631 – Evaluation of Human Exposure to Whole-Body Vibration
Vibration Measurement

• Requires high speed data acquisition
• Sample rate matters – avoid aliasing
• Sensors options:
  – Accelerometers (light/stiff)
  – Velocity transducers (e.g. geophone)
  – Displacement gages – less responsive to vibrations
Case Study:
Hospital Operating Room
Hospital Operating Room: Background

• Operating room in basement level of low rise hospital building
• Reinforced concrete slab on ground of unknown design
• Columns frame into slab
• Continuous, steady-state floor vibration throughout day
• Nuisance / distraction to doctors – potential hazard
• Vibration not perceptible elsewhere on basement level
Hospital Operating Room: Investigation

- Trace vibration “signature” to potential nearby sources
- Very sensitive (10 V/g) accelerometers used to measure motion
- Measured steady-state response of OR floor at about 20 Hz w/ amplitude \( \approx 2 \times 10^{-5} \) g
- Potential sources investigated:
  - Piping/mechanicals in sub-basement nearby
  - Portable air handlers in basement
  - HVAC units in penthouse on roof
Hospital Operating Room: Air Handlers

Operating Room

Vibration response near 20 Hz

Air Handlers

Multiple modes identified, but none at 20 Hz
Hospital Operating Room: Rooftop HVAC

Suspected A/C Unit

Rigid Mounts

Motor

Leg directly above column
Hospital Operating Room: Rooftop HVAC

Isolators Beneath Compressors
Hospital Operating Room: Rooftop HVAC

Operating Room

Rooftop AC Unit

Vibration response near 20 Hz

Primary mode also near 20 Hz
Hospital Operating Room: Findings

- A/C fan motor clearly the source of vibration in OR
- Direct connection of support over column provided clear transmission path
- Options included
  - Relocation of unit
  - Support framing modification
  - Isolation
- **Solution:** Identified appropriate isolation system for A/C unit

Can be shown that isolator stiffness, $k$, that gives
$$
\beta = \frac{\omega}{\omega_n} > \sqrt{2}
$$
results in attenuation of motion
Case Study: Chemical Process Plant
Chemical Process Plant: Background

- Initiative to improve facility & increase production
  - Increased product demand
  - Aging facility & technology
- Already frequent down time to remedy vibration related problems
- Perceived large amplitude motion of Fluidized Bed Reactors (FBRs) during production thought to be source of vibration problems
- Concerns
  - Vibration related problems would worsen with production increase
  - Potential damage to FBR supports and “Feed Materials Building”
  - No in-house engineering expertise to address the concerns
Chemical Process Plant: Fluidized Bed Reactor

Typical Fluidized Bed Reactor (FBR)
Chemical Process Plant: Scope

• Structural condition assessment
  – Are there signs of fatigue or other distress?

• Modal testing of FBRs during shutdown
  – What are the dynamic properties?

• Vibration monitoring of FBRs during operation
  – How does it behave under normal excitation?
Chemical Process Plant: Modal Testing

- Instrumented FBRs at various locations with accelerometers
- Excited FBRs with the rapid release of a tensioned chain
- Recorded acceleration data as FBR freely vibrated and eventually settled to rest position
- Estimated natural frequencies, damping, and mode shapes
Chemical Process Plant: Modal Testing

Typical Chain Release Setup

- Sling around FBR flange
- Chain hoist
- Load cell
- Lever load binder
- Sling to steel column (not in view)
- Hoop in chain
Chemical Process Plant: Modal Testing

Typical Sensor Setup

Approximate direction of chain release

FBRs had rods attached to attempt to reduce vibration

(+) acceleration in direction of arrow
(+) acceleration out of plane of page
(+) acceleration into plane of page
Chemical Process Plant: Modal Testing

Sample Free Vibration Data – Tip of FBR
Chemical Process Plant: Operational Monitoring

- Instrumented FBRs with accelerometers at same locations as modal testing
- Recorded acceleration data for 5-10 minute periods as FBRs vibrated during operation
- Estimated operational frequencies & shapes and compared to modal properties
- Computed summary statistics of response
- Computed displacement amplitudes
Chemical Process Plant: Operational Monitoring

Sample Operational Data – Tip of FBR
Chemical Process Plant: Operational Monitoring

Computed Displacements – Tip of FBRs

Operational Statistics:

<table>
<thead>
<tr>
<th></th>
<th>#1</th>
<th>#2</th>
<th>#3</th>
<th>#4</th>
<th>#5</th>
<th>#6</th>
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<td><strong>Acceleration</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>(in/s²) Peak</td>
<td>129.0</td>
<td>91.7</td>
<td>111.2</td>
<td><strong>161.7</strong></td>
<td>66.1</td>
<td>71.0</td>
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<td>RMS</td>
<td><strong>41.8</strong></td>
<td>21.1</td>
<td>29.7</td>
<td>27.1</td>
<td>18.2</td>
<td>18.1</td>
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<td><strong>Velocity</strong></td>
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<tr>
<td>(in/s) Peak</td>
<td>2.7</td>
<td>1.7</td>
<td><strong>3.3</strong></td>
<td>2.6</td>
<td>2.1</td>
<td>1.3</td>
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<td>RMS</td>
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<td>0.41</td>
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<tr>
<td>(in) Peak</td>
<td>0.12</td>
<td>0.07</td>
<td><strong>0.16</strong></td>
<td>0.11</td>
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<td>0.05</td>
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<td>RMS</td>
<td><strong>0.03</strong></td>
<td>0.02</td>
<td><strong>0.03</strong></td>
<td><strong>0.03</strong></td>
<td><strong>0.03</strong></td>
<td><strong>0.02</strong></td>
</tr>
</tbody>
</table>
Chemical Process Plant: Summary of Findings

- Low frequency pendulum-like natural modes observed during free vibration study (3-6 Hz)
- Vessel vibrations centered at similar frequencies during operation – excitation fairly broadband
- Amplitudes of displacements small (+/- 0.1 inch)
- Additional high frequency vibrations transmitted from surroundings (60 Hz)
- **Conclusion:** Non-resonant response; perception worse than reality
Chemical Process Plant: Recommendations

- Further evaluate fatigue potential of support lugs / framing
  - Stress analysis based on operational data
- External damping devices on FBRs
  - Provided concept for tuned mass damper (vibration absorber) and viscous dampers
- Isolate other machinery to reduce vibration transmission, noise, and maintenance issues
Case Study: Theater Balcony
Theater Balcony: Background

- 100-year-old historic theater
- Major renovations 30 years ago
- Concert events now range from symphony to rock to hip-hop
- Concert goer experienced discomforting movement of the balcony during a lively concert
- Incident reported to the city building department
- City required owner to evaluate
- Owner concerned about potential for structural damage
Theater Balcony: Cross Section
Theater Balcony: Instrumentation

Accelerometer #1
Accelerometer #2
Accelerometer #3
Theater Balcony: Acceleration Data

Accelerometer #1 (Left)

Accelerometer #2 (Middle)

Accelerometer #3 (Right)
Theater Balcony: Dynamic Properties

- Natural frequency of empty balcony \( \approx 4.5 \) Hz
- Natural frequency of full balcony \( \approx 3.5 \) Hz (added mass)
- Frequency of synchronized jumping crowds = 1.8-2.3 Hz
- No signs of adverse dynamic amplification from audience movement
Theater Balcony: Serviceability Criteria

- Peak Computed Displacement $\approx 1$ in
- Typical Serviceability Limit (L/360) $\approx 2$ in
- Response Statistics
  - Mean + 3 Std. Dev. $\leq 0.15$ in
  - More than 99% of response is *much* less than serviceability limits
Theater Balcony: Energy-Based Criteria

- **Human/Structural Response**
  - Threshold, minor cosmetic damage fragile buildings

- **Velocity Level**
  - 100

- **Typical Sources (50 ft from source)**
  - Blasting from construction projects
  - Bulldozers and other heavy tracked construction equipment
  - Commuter rail, upper range
  - Rapid transit, upper range
  - Commuter rail, typical
  - Bus or truck over bump
  - Rapid transit, typical
  - Bus or truck, typical

- **Typical Background Vibration**

- **Frequency (Hz)**

- **Velocity (VdB)**

- **Graph**
  - Left Location
  - Middle Location
  - Right Location

* RMS Vibration Velocity Level in VdB relative to $10^{-6}$ inches/second
Theater Balcony: Findings

- Displacements generally small – no sign of adverse dynamic amplification
- Borderline risk for cosmetic damage based on energy criteria
- But, no evidence of cosmetic damage to sensitive brittle finishes or damage to underlying steel framing
- Vibration levels could be uncomfortable – outside a concert setting
- Conclusion: a sensitive patron signaled a false alarm
Wrap Up
Summary

• SDOF model is the most basic tool for structural vibration analysis
• Natural frequency – related to stiffness/mass – is a key parameter
• Resonance occurs when excitation frequency is at/near natural frequency – this is bad news
• Analysis approach depends heavily on nature of forcing function – dependent on vibration source
• People have fairly sensitive vibration filters – much more than structures
Acknowledgements

- Iowa Section ASCE
- Paul Thompson
- Stephen Jones
- Jennifer Vit
Questions?