Ag Decision Maker

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Understanding the Time Value of Money

If I offered to give you $100, you would probably say yes. Then, if I asked you if you wanted the $100 today or one year from today, you would probably say today. This is a rational decision because you could spend the money now and get the satisfaction from your purchase now rather than waiting a year. Even if you decided to save the money, you would rather receive it today because you could deposit the money in a bank and earn interest on it over the coming year. So there is a time value to money.

Next, let's discuss the size of the time value of money. If I offered you $100 today or $105 dollars a year from now, which would you take? What if I offered you $110, $115, or $120 a year from today? Which would you take? The time value of money is the value at which you are indifferent to receiving the money today or one year from today. If the amount is $115, then the time value of money over the coming year is $15. If the amount is $110, then the time value is $10. In other words, if you will receive an additional $10 a year from today, you are indifferent to receiving the money today or a year from today.

When discussing the time value of money, it is important to understand the concept of a time line. Time lines are used to identify when cash inflows and outflows will occur so that an accurate financial assessment can be made. A time line is shown in the next column with five time periods. The time periods may represent years, months, days, or any length of time so long as each time period is the same length of time. Let's assume they represent years. The zero tick mark represents today. The one tick mark represents a year from today. Any time during the next 365 days is represented on the time line from the zero tick mark to the one tick mark. At the one tick mark, a full year has been completed. When the second tick mark is reached, two full years have been completed, and it represents two years from today. You can move on to the five tick mark, which represents five years from today.

Time Line

Because money has a time value, it gives rise to the concept of interest. Interest can be thought of as rent for the use of money. If you want to use my money for a year, I will require that you pay me a fee for the use of the money. The size of the rental rate or user fee is the interest rate. If the interest rate is 10 percent, then the rental rate for using $100 for the year is $10.

Compounding

Compounding is the impact of the time value of money (e.g., interest rate) over multiple periods into the future, where the interest is added to the original amount. For example, if you have $1,000 and invest it at 10 percent per year for 20 years, its value after 20 years is $6,727. This assumes that you leave the interest amount earned each year with the investment rather than withdrawing it. If you remove the interest amount every year, at the end of 20 years the $1,000 will still be worth only $1,000. But if you leave it with the investment, the size of the investment will grow exponentially. This is because you are earning interest on your interest. This process is called compounding. And, as the amount grows, the size of the interest amount will also grow. As shown in Table 1, during the first year of a $1,000 investment, you will earn $100 of interest if the interest rate is 10 percent. When the $100 interest is added to the $1,000 investment it becomes $1,100 and 10 percent of $1,100 in year two is $110. This process continues until year 20, when the amount of interest is 10 percent of $6,116 or $611.60. The amount of the investment is $6,727 at the end of 20 years.

The impact of compounding outlined in Table 1 is

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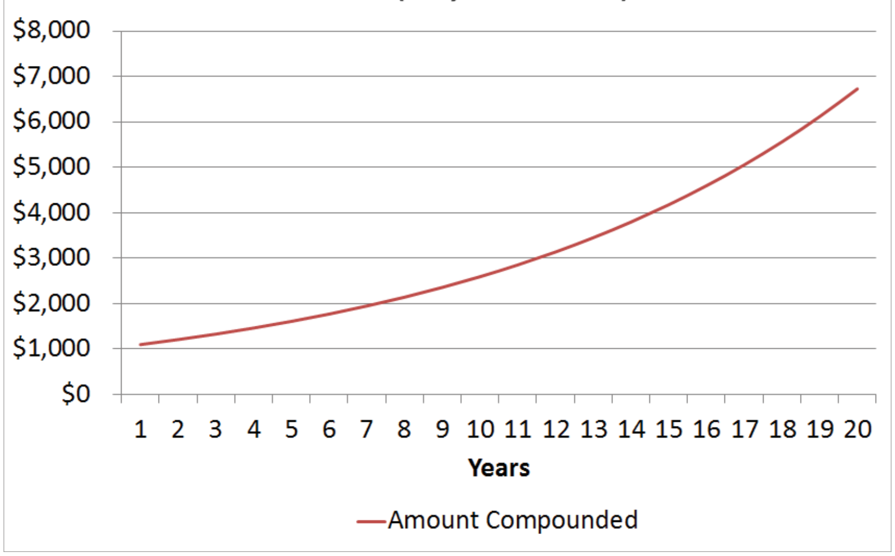
shown graphically in Figure 1. The increase in the size of the cash amount over the 20-year period does not increase in a straight line but rather exponentially. Because the slope of the line increases over time it means that each year the size of the increase is greater than the previous year. If the time period is extended to 30 or 40 years, the slope of the line would continue to increase. Over the long-term, compounding is a very powerful financial concept.

The effect of compounding is also greatly impacted by the size of the interest rate. Essentially, the larger the interest rate the greater the impact of compounding. In addition to the compounding impact of a 10 percent interest rate, Figure 2 shows the impact of a 15 percent interest rate (5 percent higher rate) and 5 percent (5 percent lower rate). Over the 20-year period, the 15 percent rate yields almost $10,000 more than the 10 percent rate, while the 5 percent rate results in about $4,000 less. By examining the last 10 years of the 20-year period you can see that increasing the number of time periods and the size of the interest rate greatly increases the power of compounding.

Table 1. Compounding computation of $1,000 over 20 years at an annual interest rate of 10 percent.

Year	Amount	Computation
0	$1,000	
1	$1,100	$1,000 x 10% = $100 + $1,000 = $1,100
2	$1,210	$1,100 x 10% = $110 + $1,100 = $1,210
3	$1,331	$1,210 x 10% = $121 + $1,210 = $1,331
4	$1,464	$1,331 x 10% = $133 + $1,331 = $1,464
5	$1,611	$1,464 x 10% = $146 + $1,464 = $1,611
6	$1,772	$1,611 x 10% = $161 + $1,611 = $1,772
7	$1,949	$1,772 x 10% = $177 + $1,772 = $1,949
8	$2,144	$1,949 x 10% = $195 + $1,949 = $2,144
9	$2,358	$2,144 x 10% = $214 + $2,144 = $2,358
10	$2,594	$2,358 x 10% = $236 + $2,358 = $2,594
11	$2,853	$2,594 x 10% = $259 + $2,594 = $2,853
12	$3,138	$2,853 x 10% = $285 + $2,853 = $3,138
13	$3,452	$3,138 x 10% = $314 + $3,138 = $3,452
14	$3,797	$3,452 x 10% = $345 + $3,452 = $3,797
15	$4,177	$3,797 x 10% = $380 + $3,797 = $4,177
16	$4,959	$4,177 x 10% = $418 + $4,177 = $4,595
17	$5,054	$4,595 x 10% = $456 + $4,595 = $5,054
18	$5,560	$5,054 x 10% = $505 + $5,054 = $5,560
19	$6,116	$5,560 x 10% = $556 + $5,560 = $6,116
20	$6,727	$6,116 x 10% = $612 + $6,116 = $6,727

Figure 1. Impact of Compound Interest (20 years at 10%)



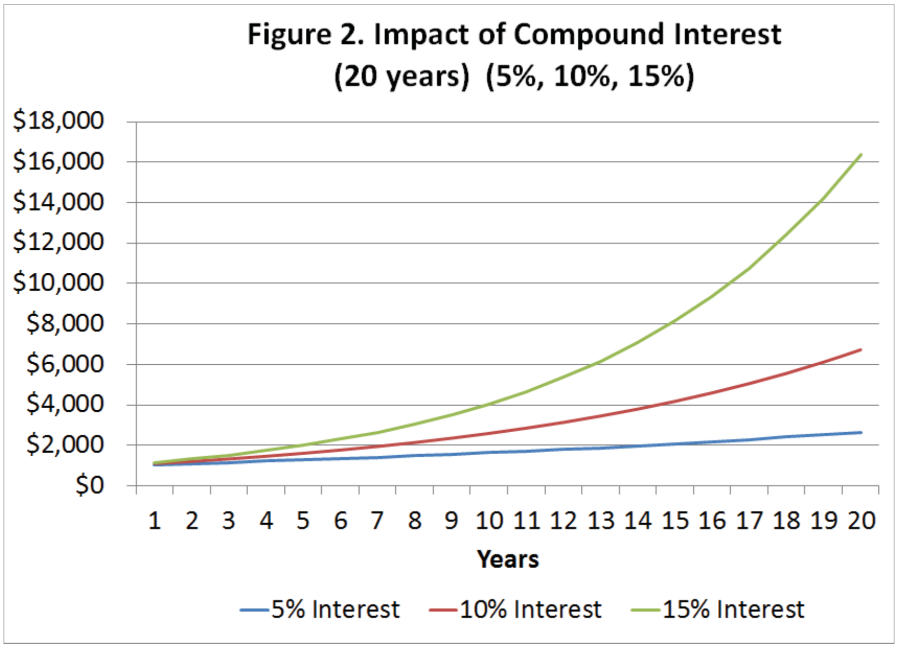


Table 2. Compounding computation of $1,000 over 20 years at an annual interest rate of 10 percent.

Year	Amount	Computation
0	$1,000	
1	$1,100	$1,000 x 1.10 = $1,100
2	$1,210	$1,100 x 1.10 = $1,210
3	$1,331	$1,210 x 1.10 = $1,331
4	$1,464	$1,331 x 1.10 = $1,464
5	$1,611	$1,464 x 1.10 = $1,611
6	$1,772	$1,611 x 1.10 = $1,772
7	$1,949	$1,772 x 1.10 = $1,949
8	$2,144	$1,949 x 1.10 = $2,144
9	$2,358	$2,144 x 1.10 = $2,358
10	$2,594	$2,358 x 1.10 = $2,594
11	$2,853	$2,594 x 1.10 = $2,853
12	$3,138	$2,853 x 1.10 = $3,138
13	$3,452	$3,138 x 1.10 = $3,452
14	$3,797	$3,452 x 1.10 = $3,797
15	$4,177	$3,797 x 1.10 = $4,177
16	$4,595	$4,177 x 1.10 = $4,595
17	$5,054	$4,595 x 1.10 = $5,054
18	$5,560	$5,054 x 1.10 = $5,560
19	$6,116	$5,560 x 1.10 = $6,116
20	$6,727	$6,116 x 1.10 = $6,727

Table 2 shows the same compounding impact as Table 1 but with a different computational process. An easier way to compute the amount of compound interest is to multiply the investment by one plus the interest rate. Multiplying by one maintains the cash amount at its current level and .10 adds the interest amount to the original cash amount. For example, multiplying $1,000 by 1.10 yields $1,100 at the end of the year, which is the $1,000 original amount plus the interest amount of $100 ($1,000 x .10 = $100).

Another dimension of the impact of compounding is the number of compounding periods within a year. Table 3 shows the impact of 10 percent annual compounding of $1,000 over 10 years. It also shows the same $1,000 compounded semiannually over the 10-year period. Semiannual compounding means a 10 percent annual interest rate is converted to a 5 percent interest rate and charged for half of the year. The interest amount is then added to the original amount, and the interest during the last half of the year is 5 percent of this larger amount. As shown in Table 3, semiannual compounding will result in 20 compounding periods over a 10-year period, while annual compounding results in only 10 compounding period. A shorter compounding period means a larger number of compounding periods over a given time period and

Table 3. A comparison of $1,000 compounding annually versus semiannually (20 years, 10 percent interest)

	Compounding Annually		Compounding Semiannually	
Year	Amount	Computation	Amount	Computation
0	$1,000		$1,000	
0.5			$1,050	$1,000 x 1.05 = $1,050
1	$1,100	$1,000 x 1.10 = $1,100	$1,103	$1,050 x 1.05 = $1,103
1.5			$1,158	$1,103 x 1.05 = $1,158
2	$1,210	$1,100 x 1.10 = $1,210	$1,216	$1,158 x 1.05 = $1,216
2.5			$1,276	$1,216 x 1.05 = $1,276
3	$1,331	$1,210 x 1.10 = $1,331	$1,340	$1,276 x 1.05 = $1,340
3.5			$1,407	$1,276 x 1.05 = $1,407
4	$1,464	$1,331 x 1.10 = $1,464	$1,477	$1,407 x 1.05 = $1,477
4.5			$1,551	$1,477 x 1.05 = $1,551
5	$1,611	$1,464 x 1.10 = $1,611	$1,629	$1,551 x 1.05 = $1,629
5.5			$1,710	$1,629 x 1.05 = $1,710
6	$1,772	$1,611 x 1.10 = $1,772	$1,796	$1,710 x 1.05 = $1,796
6.5			$1,886	$1,710 x 1.05 = $1,886
7	$1,949	$1,772 x 1.10 = $1,949	$1,980	$1,886 x 1.05 = $1,980
7.5			$2,079	$1,980 x 1.05 = $2,079
8	$2,144	$1,949 x 1.10 = $2,144	$2,183	$2,079 x 1.05 = $2,183
8.5			$2,292	$2,183 x 1.05 = $2,292
9	$2,358	$2,144 x 1.10 = $2,358	$2,407	$2,292 x 1.05 = $2,407
9.5			$2,527	$2,407 x 1.05 = $2,527
10	$2,594	$2,358 x 1.10 = $2,594	$2,653	$2,527 x 1.05 = $2,653

a greater compounding impact. If the compounding period is shortened to monthly or daily periods, the compounding impact will be even greater.

Discounting

Although the concept of compounding is straight forward and relatively easy to understand, the concept of discounting is more difficult. However, the important fact to remember is that discounting is the opposite of compounding. As shown below, if we start with a future value of $6,727 at the end of 20 years in the future and discount it back to today at an interest rate of 10 percent, the present value is $1,000

Table 4. Discounting computation of $6,727 over 20 years at an annual discount rate of 10 percent.

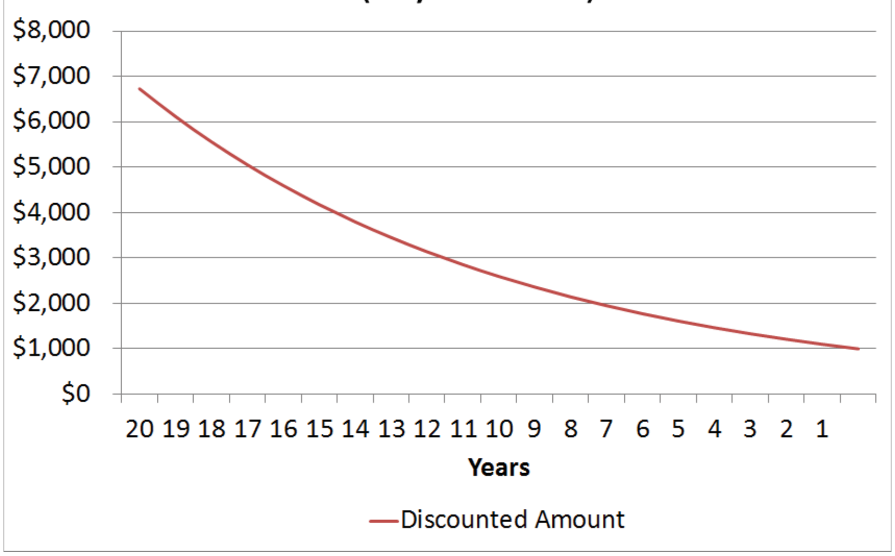
Year	Amount	Computation
20	$6,727	
19	$6,116	$6,727 x .91 = $6,116
18	$5,560	$6,115 x .91 = $5,560
17	$5,054	$5,560 x .91 = $5,054
16	$4,595	$5,054 x .91 = $4,595
15	$4,177	$4,595 x .91 = $4,177
14	$3,797	$4,117 x .91 = $3,797
13	$3,452	$3,797 x .91 = $3,452
12	$3,138	$3,452 x .91 = $3,138
11	$2,853	$3,138 x .91 = $2,853
10	$2,594	$2,853 x .91 = $2,594
9	$2,358	$2,594 x .91 = $2,358
8	$2,144	$2,358 x .91 = $2,144
7	$1,949	$2,144 x .91 = $1,949
6	$1,772	$1,949 x .91 = $1,772
5	$1,611	$1,772 x .91 = $1,611
4	$1,464	$1,661 x .91 = $1,464
3	$1,331	$1,464 x .91 = $1,331
2	$1,210	$1,331 x .91 = $1,210
1	$1,100	$1,210 x .91 = $1,100
0	$1,000	$1,100 x .91 = $1,000

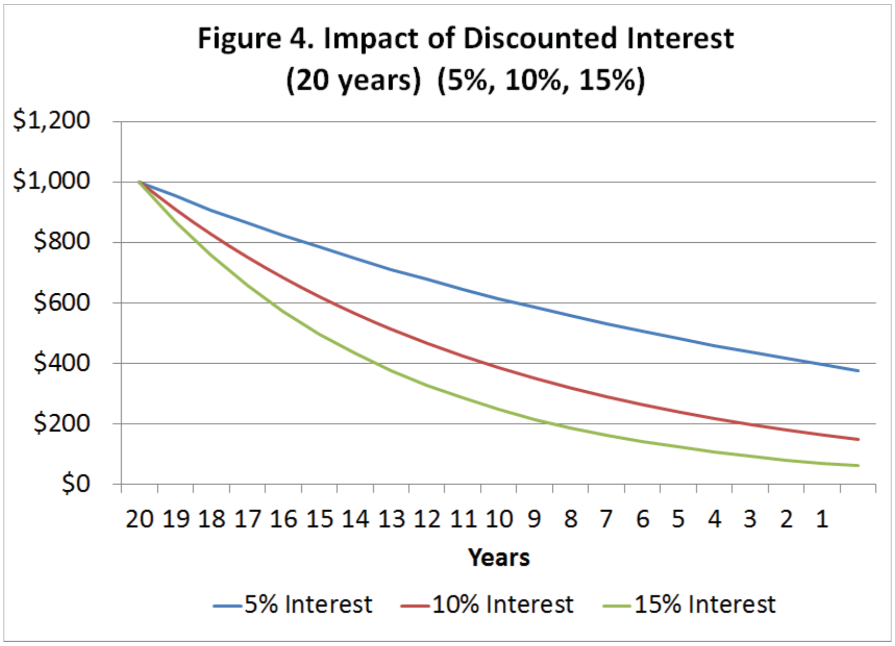
As shown in Table 2, the compounding factor of annually compounding at an interest rate of 10 percent is 1.10 or 1.10/1.00. If discounting is the opposite of compounding, then the discounting factor is 1.00 / 1.10 = .90909 or .91. As shown in Table 4, the discounted amount becomes smaller as the time period moves closer to the current time period. When we compounded $1,000 over 20 years at a 10 percent interest rate, the value at the end of the period is $6,727 (Table 2). When we discount $6,727 over 20 years at a 10 percent interest rate, the present value or value today is $1,000.

The discounting impact is shown in Figure 3. Note that the curve is the opposite of the compounding curve in Figure 1.

The impact of discounting using interest rates of 5 percent, 10 percent, and 15 percent is shown in Figure 4. The 15 percent interest rate results in a larger discounting impact than the 10 percent rate, just as the 15 percent interest rate results in a larger compounding impact as shown in Figure 2.

Figure 3. Impact of Discounted Interest (20 years at 10%)





Discounting Example

An example of discounting is to determine the present value of a bond. A bond provides a future stream of income. It provides a cash return at a future time period, often called the value at maturity. It may also provide a stream of annual cash flows until the maturity of the bond. Table 5 shows an example of a $10,000 bond with a 10-year maturity. In other words, the bond will yield $10,000 at maturity, which is received at the end of 10 years. The bond also has an annual annuity (an annuity is a stream of equal cash payments at regular time intervals for a fixed period of time) equity to 10 percent of the value at maturity. So, the bond yields 10 $1,000 (10% x $10,000) annual payments over the 10-year period. Adding together the 10 $1,000 payments plus the $10,000 value at maturity, the future cash return from the bond is $20,000.

Table 5. 10-year bond with 10 percent annual annuity and $10,000 payout at the end of 10 years.

Year	Annuity	Value at Maturity	Total
0			
1	$1,000		$1,000
2	$1,000		$1,000
3	$1,000		$1,000
4	$1,000		$1,000
5	$1,000		$1,000
6	$1,000		$1,000
7	$1,000		$1,000
8	$1,000		$1,000
9	$1,000		$1,000
10	$1,000	$10,000	$11,000
Total	**$10,000**	**$10,000**	**$20,000**

To compute the current value of the bond, we must discount the future cash flows back to the time when the bond is purchased. To do this we must select an interest rate (called the discount rate when we are discounting). In Table 6, we have calculated the present value of the bond using discount rates of 5 percent, 10 percent, and 15 percent. First, let's examine the computation using a 5 percent rate. Each of the $1,000 annuity payments is discounted to the present value. Note that the one year $1,000 annuity payment has a present value of $952 and the 10-year payment has a present value of $614. This is because the first-year payment is only discounted one time and the tenth-year payment is discounted 10 times over 10 years. The present value of all 10 annuity payments is $7,722. The present value of the

$10,000 at maturity (after 10 years) is $6,139. Note that the present value of the $10,000 of annual annuity payments is greater than the $10,000 payment received at maturity because most of the annuity payments are discounted over time periods less than 10 years. The total present value of the annuity and the value at maturity is $13,861. So, the $20,000 of future cash payments has a value at the time of purchase of $13,861. Looking at it from a different perspective, if you paid $13,861 for this bond you would receive a 5 percent annual rate of return (called the internal rate of return) over the 10-year period.

Table 6. Computing the Present Value of the Bond Shown in Table 5.

Discount Rate = 5 Percent

Year	Annuity	Value at Maturity	Total
0			
1	$952		$952
2	$907		$907
3	$864		$864
4	$823		$823
5	$784		$784
6	$746		$746
7	$711		$711
8	$677		$677
9	$645		$645
10	$614	$6,139	$6,753
Total	**$7,722**	**$6,139**	**$13,861**

If we increase the discount rate from 5 percent to 10 percent, the discounting power becomes greater. The present value of the bond drops from $13,861 to $10,000. In other words, if you want a 10 percent rate of return you can only pay $10,000 for the bond that will generate $20,000 in future cash payments. Note that the value at maturity dropped over $2,000 from $6,139 to $3,855. Conversely, the value of the annuity dropped from $7,722 to $6,145, a reduction of about $1,600.

Discount Rate = 10 Percent

Year	Annuity	Value at Maturity	Total
0			
1	$909		$909
2	$826		$826
3	$751		$751
4	$683		$683
5	$621		$621
6	$564		$564
7	$513		$513
8	$467		$467
9	$424		$424
10	$386	$3,855	$4,241
Total	**$6,145**	**$3,855**	**$10,000**

If we increase the discount rate to 15 percent, the discounting power becomes even greater. The present value of the bond drops to $7,491. In other words, if you want a 15 percent rate of return you can only pay $7,491 for the bond that will generate $20,000 in future cash payments. Note that the value at maturity dropped from $6,139 (5 percent) to $3,855 (10 percent) to $2,472 (15 percent). The value of the annuity dropped in smaller increments from $7,722 (5 percent) to $6,145 (10 percent) to $5,019 (15 percent).

Discount Rate = 15 Percent

Year	Annuity	Value at Maturity	Total
0			
1	$870		$870
2	$756		$756
3	$658		$658
4	$572		$572
5	$497		$497
6	$432		$432
7	$376		$376
8	$327		$327
9	$284		$284
10	$247	$2,472	$2,719
Total	**$5,019**	**$2,472**	**$7,491**

A bond is a simple example of computing the present value of an asset with an annual cash income stream and a terminal value at the end of the time period. This methodology can be used to analyze any investment that has an annual cash payment and a terminal or salvage value at the end of the time period.

Perpetuity

A perpetuity is similar to an annuity except that an annuity has a limited life and a perpetuity is an even payment that has an unlimited life. The computation of a perpetuity is straight forward. The present value of a perpetuity is the payment divided by the discount rate.

$$\text{Present Value} = \frac{\text{Perpetuity Payment}}{\text{Discount Rate}}$$

Payment = $10,000
Discount Rate = 10 percent
Present Value = ?

$$\frac{\$10{,}000}{10\%} = \$100{,}000$$

Time Value of Money Formulas

There are mathematical formulas for compounding and discounting that simplify the methodology. At right are the formulas, in which:

- "PV" represents the present value at the beginning of the time period.
- "FV" represents the future value at the end of the time period.
- "N" or "Nper" represents the number of compounding or discounting periods. It can represent a specific number of years, months, days, or other predetermined time periods.
- "Rate" or "i" represents the interest rate for the time period specified. For example, if "N" represents a specified number of years, then the interest rate represents an annual interest rate. If "N" represents a specific number of days, then the interest rate represents a daily interest rate.

If we are computing the compounded value of a current amount of money into the future, we will use the following formula. The future value "FV" that we are solving for is the current amount of money "PV" multiplied by one plus the interest rate to the power of the number of compounding periods. We are solving for the future compounded value (FV), in which the present value (PV) is $1,000, the annual interest rate (Rate) is 10 percent, and the number of time periods (Nper) is 20 years. This results in $1,000 multiplied by 6.727 and a future value of $6,727. Note that this is the same result as shown in Tables 1 and 2.

$$FV = PV\ (1 + Rate)^{Nper}$$

$$FV = \$1{,}000\ (1 + .10)^{20}$$

$$FV = \$1{,}000 \times 1.10^{20}$$

$$FV = \$1{,}000 \times 6.727$$

$$FV = \$6{,}727$$

To compute the discounted value of an amount of money to be received in the future, we use the same formula but solve for the present value rather than the future value. To adjust our formula, we divided both sides by $(1 + Rate)^{Nper}$ and the following formula emerges.

$$PV = \frac{FV}{(1 + Rate)^{Nper}}$$

$$PV = \frac{\$6{,}727}{(1 + .10)^{20}}$$

$$PV = \frac{\$6{,}727}{1.10^{20}}$$

$$PV = \frac{\$6{,}727}{6.727}$$

$$PV = \$1{,}000$$

The present value (PV) of a future value (FV) of $6,727 discounted over 20 years (Npers) at an annual discount interest rate (Rate) of 10 percent is $1,000, the same as shown in Table 4.

Time Value of Money Computation

A financial calculator or an electronic spreadsheet on a personal computer is a useful tool for making time value of money computations. For compounding computations, you enter the present value, interest rate, and the number of time periods, and the calculator or personal computer will compute the future value. The future value for the example below is $6,727, the same as the future value shown in Tables 1 and 2.

Present Value (PV) = $1,000
Interest Rate (Rate) = 10 percent
Number of Periods (Nper) = 20 years
Future Value (FV) = ?

Likewise, for discounting computations you enter the future value, interest rate, and the number of time periods, and the calculator or personal computer will compute the present value. The present value for the example below is $1,000, the same as the present value shown in Table 4.

Future Value (FV) = $6,727
Discount Rate (Rate) = 10 percent
Time Period (Nper) = 20 years
Present Value (PV) = ?

If an annuity is involved, you can use the payment function (PMT). In the example below, the present value is $10,000, the same as the present value of the bond example in Table 6.

Future Value (FV) = $10,000
Discount Interest Rate (Rate) = 10 percent
Time Period (Nper) = 10 years
Annuity (PMT) = $1,000
Present Value (PV) = ?

By using a financial calculator or personal computer, you can compute any of the values in the examples above as long as you know the other values. For example, you can compute the interest rate if you know the future value, present value, and number of time periods. You can compute the number of time periods if you know the present value, future value, and interest rate. The same is true if an annuity is involved.

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